# Section 5.5 Notes — Related Rates

*(Basically relates functions to each other)

i.e. $\left(\frac{d}{dt}\right)$

Example: $\frac{d}{dt}$ $\quad xy + y^2 = 7x$

Product Rule, so pull out & solve

Step 1

1st piece $xy$ $\quad$ (+) $\quad$ 2nd piece $y^2$ $\quad$ (=) $\quad$ 3rd piece $7x$

$f = x \quad g = y$ $\qquad + 2y \cdot \frac{dy}{dt}$ $\qquad 7 \cdot \frac{dx}{dt}$

Step 2

$f' = \frac{dx}{dt} \quad g = \frac{dy}{dt}$

Step 3

reassemble into formula

$f'g + g'f$

1st piece: $\frac{dx}{dt}(y) + x\left(\frac{dy}{dt}\right)$ $\qquad$ 2nd piece: $2y \cdot \frac{dy}{dt}$ $\qquad$ 3rd piece: $\left(7 \cdot \frac{dx}{dt}\right)$

Step 4 Combine 3 pieces:

$$\underbrace{y \cdot \frac{dx}{dt} + x\frac{dy}{dt} + 2y \cdot \frac{dy}{dt} = 7\frac{dx}{dt}}$$

Put $y$, #, $x$ in front of $\frac{dx}{dt}$.

Ex: $\frac{d}{dz}$ $\underbrace{x^2y^3}_{\text{Product Rule}} + \underbrace{e^y}_{\text{single derivative}} = \underbrace{3}_{\text{single derivative}}$

Step 1

Step 2 1st piece + 2nd piece = 3rd piece

$x^2y^3$ $\quad e^y \quad 3$

Step 3

$f = x^2$ $\quad g = y^3$ $\quad = e^y \cdot \frac{dy}{dz}$ $\quad \varnothing$

$f' = 2x \cdot \frac{dx}{dz}$ $\quad g' = 3y^2 \cdot \frac{dy}{dz}$

Step 4 1st piece into formula $f'g + g'f$

$2x \cdot \frac{dx}{dz} (y^3) + 3y^2 \cdot \frac{dy}{dz} \cdot x^2$

$= 2xy^3 \cdot \frac{dx}{dz} + 3x^2y^2 \cdot \frac{dx}{dz}$

1st Piece + 2nd Piece = 3rd Piece

$$\boxed{2xy^3 \cdot \frac{dx}{dz} + 3x^2y^2 \cdot \frac{dy}{dz} + e^y \cdot \frac{dy}{dz} \qquad \varnothing}$$

Final answer

# #10 Practice

~~3~~ 4 Step Process w/ word problems

1) Find an equation (often come from geometry)

2) Determine our goal (what do we need to solve for?)

3) Set up the equation & take its derivative (often WRT (t) $\frac{d?}{dt}$)

4) Answer the indicated question

#10 Step 1 Goal: How fast is area growing, so $\cancel{\frac{da}{dt}}$ $\frac{dA}{dt}$

Step 2 Need $A = \ldots\ldots$
Looking for area of a circle, which is

$A = \pi r^2$ ⟶ also similar to $3x^2$; $\pi$ just a #

Step 3 Find $\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}$

Step 4 Look @ problem again & see what #'s it gives us.

Hint: Growing/Changing = 1st derivative, so if growing @ 5cm ± $\frac{dr}{dt} = +5$ cm/s

Step5 $\frac{dA}{dt}$ = how fast area growing (Goal!)
b/c A = Area

Step6 Formula: $\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}$

we already know $\frac{dr}{dt} = 5 cm/s$

now problem asking us when
r = 10 cm what is Area?

$r = 10 cm$

$\frac{dr}{dt} = +5 cm/s$

Plug into equation $\frac{dA}{dt} = 2\pi(10)(5)$

$\boxed{= 100\pi \ cm^2/s}$

Note: Area = squared = $A^2$ = always
volume = always = cubed = $V^3$

Part b

Step 1 Goal: Growing = $\frac{dA}{dt}$

need (r) or $\frac{dr}{dt}$ ✓

Step 2 it = $36 \ cm^2$
Is Area b/c squared

Step3 $A \pi r^2$

Step 4 $36 = \pi r^2$

Step5 $\frac{36}{\pi} = r^2$

Step6 $\sqrt{\frac{36}{\pi}} = r$

Step7 $5 \ cm/s = \frac{dr}{dt}$

$\boxed{\text{Step 8}}$ $\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}$

$\boxed{\text{Step 9}}$ $\frac{dA}{dt} = 2\pi\left(\sqrt{\frac{36}{\pi}}\right)(5)$

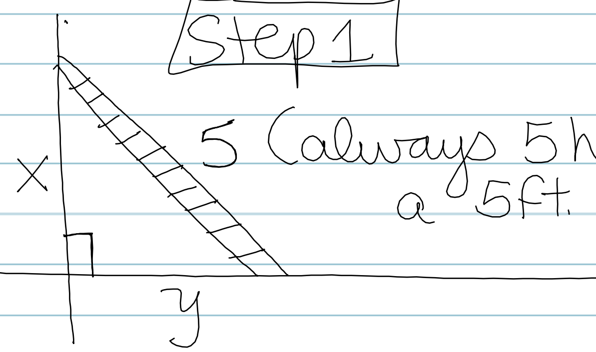
$= 106.35 \underbrace{cm^2/s}$

Have to put it in as units of measurement

Hint: look @ clues & units given.

Example #14

$\boxed{\text{Step 1}}$




5 (always 5 here b/c is a 5ft. ladder)

Step 2 Pythagoreum Theorum

$a^2 + b^2 = c^2$

, so $x^2 + y^2 = (5)^2$

$\boxed{x^2 + y^2 = 25}$ Equation deals w/ #'s only & physical positions on wall.

Step 3 Find derivative w/ respect to (time)

$\boxed{2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} = 0}$ Equation deals with how its moving & in what direction.

$\boxed{\text{Step 4}}$ down the wall $\frac{dx}{dt} = -10$ / ft per second or $-10$ ft/s

$\boxed{\text{Step 5}}$ away from wall $\frac{dy}{dt} =$

$x = 3$ b/c 3ft away from wall and $y = ?$

Goal

Step 5 $\overset{3}{2x} \cdot \frac{\overset{-10}{dx}}{dt} + 2y \cdot \frac{dy}{dt} = \emptyset$

$x^2 + y^2 = 25$ * missing (y) value, so use original equation to solve for y.

Step 6 $x^2 + y^2 = 25 \quad x = 3; \quad (3)^2 + y^2 = 25$

$-9 \qquad -9$

$y^2 = 16$

$y = 4$

Step 7 $2(3)(-10) + 2(4)\frac{dy}{dt} = \emptyset; \quad -60 + 8\frac{dy}{dt} = \emptyset;$

$= 8\frac{dy}{dt} = 60; \quad = \frac{dy}{dt} = \frac{60}{8};$

Step 8 $\boxed{\frac{dy}{dt} = 7.5 \text{ ft/s}}$